



Engineering Studies Preparation Course

Math Basics

Ratios

Ratios compare values.

They tell you how much there is of one thing, compared to another thing, or even several things.

The common notation is:

$$x : y$$

Or

$$\frac{x}{y}$$

Where x and y are both quantities of something **in the same units**

Ratios

Ratios should always be given in the simplest form which can be obtained by dividing by any common factors.

This is similar to simplifying fractions

$$\frac{2}{4} = \frac{1}{2} = \frac{10}{20} \text{ etc.}$$

$$3:6 = 1:2 = 50:100 \text{ etc.}$$

Converting Ratios to Fractions

Example: There are 3 boys and 12 girls in a class.

Ratio:

$$3:12$$

$$1:4$$

Fraction:

The fractions can be found directly from the question since it can be seen that there are 15 people in the class.

$$\text{Boys: } \frac{3}{15} = \frac{1}{5}$$

$$\text{Girls: } \frac{12}{15} = \frac{4}{5}$$

Or from the ratio:

$$\text{Boys: } \frac{1}{4+1} = \frac{1}{5}$$

$$\text{Girls: } \frac{4}{4+1} = \frac{4}{5}$$

Ratios

It is common to need to divide something according to a certain ratio. To do this, carry out the following procedure:

- 1) Specify the ratio
- 2) Write the ratio in terms of fractions
- 3) Find each fraction of the quantity (multiply the fraction by the total)

Ratios - Example

Example: You and your friends rob a bank, you decide to split the \$160 000 between the three of you according to how many guns you have. Hard has 2 guns, Killer has 5, and Rascal only has 1. How much money do they each claim.

Ratio:

$$R: H: K$$
$$1: 2: 5$$

Fraction:

$$\frac{1}{8} : \frac{2}{8} : \frac{5}{8}$$

Ratios - Example

Killer's Share:

$$\frac{5}{8} \times 160000 = \$100000$$

Hard's Share:

$$\frac{2}{8} \times 160000 = \$40000$$

Rascal's Share:

$$\frac{1}{8} \times 160000 = \$20000$$

Rates

Ratios compare quantities of the **same units**

Rates compare quantities of **different units**, and are usually expressed as fractions.

A **unit rate** is a rate that has been simplified by writing the rate as a fraction with a denominator of 1.

Rates - Example

Example: You're happily taking your girlfriend on a romantic hot-air balloon ride, when her jealous, marine ex-boyfriend snipers the balloon. In 14 seconds you notice that the balloon has collapsed to $\frac{3}{4}$ of it's original volume.

The balloon originally had a volume of 1000m^3 . Find the unit rate of the deflation of the balloon.

Rates - Example

Solution:

The volume decreases to $\frac{3}{4}$ of its initial value, so $1000 \times \frac{3}{4} = 750 \text{m}^3$ remains. Therefore 250m^3 has evacuated in the 14s.

$$\text{Rate} = \frac{250 \text{m}^3}{15 \text{s}} = \frac{250}{15} \text{m}^3 / \text{s} = 16.\dot{6} \text{m}^3 / \text{s}$$

(The dot over the 6 represents the recursive decimal)

Mean

The mean is simply the average of a set of quantities.

If you want to get technical it's a measure of central tendency.

$$\bar{x} = \frac{\sum x}{n}$$

So the mean is the sum of all quantities divided by the number of terms.

Mean

Example:

Calculate the mean of the following test scores obtained from a small class:

93%, 62%, 54%, 75%, 88%, 81%, 76%, 60%

$$\begin{aligned}\bar{x} &= \frac{\sum x}{n} \\ &= \frac{93 + 62 + 54 + 75 + 88 + 81 + 76 + 60}{8} \\ &= 73.625\end{aligned}$$

Standard Deviation

The mean of a data set is crucial in interpreting the results.

But say we had the results of two tests, both showed an average of 75%, but the data for one was:

75%, 75%, 75%, 75%

And the data for the other was:

5%, 95%, 100%, 100%

Very different conclusions can be drawn from the data, despite them having the same mean.

Standard Deviation

The second critical piece of information is the **standard deviation**.

This gives a measure of the spread or **dispersion** of the results.

So in the example in the previous slide, there is no deviation, meaning the results were totally consistent. The second lot of data on the other hand shows an outlier in the data set, there is a large deviation between the scores.

Algebra

What is the use of Algebra?

Algebra is the branch of mathematics that deals with numbers and their relations and is used throughout people's daily lives, from buying groceries in the store, to computer programs, to workplace industry and to scientific researches.

Algebra

Algebra deals mainly with numbers and variables.

- There are two types of numbers.
 - Real Numbers (Includes Rational & Irrational numbers)
 - Imaginary Numbers
- When doing algebra, variables such as x , y and z are used in equations formed on these numbers.
- In this topic, we will be learning the algebra based on real numbers. The imaginary numbers will be dealt with later in Presentation 8.

Some Definitions

Linear equation:

A mathematical expression that has an equal sign and a linear expression

Linear expression:

A mathematical statement that performs functions of addition, subtraction, multiplication, and division

Variables in linear expressions:

- All variables are to the power of 1
- No product of variables

Variable:

A number that you don't know, most commonly represented by x or y

One Variable

When solving a linear equation, the operations that are “least bound” to the variable should be removed first.

A simple way of determining what is least bound to a variable is to reverse the order of operation (BIMDAS).

BIMDAS (BIMA)

Calculating expressions
(order of calculation):

B – Brackets

I – Indices

M – Multiplication

D – Division

A – Addition

S – Subtraction

One Variable - Example

$$\frac{3 + 2f}{5} = 1$$

$$3 + 2f = 5 \times 1 = 5$$

$$2f = 5 - 3 = 2$$

$$f = \frac{2}{2} = 1$$

$$\therefore f = 1$$

Checking Solutions - Example

You're sitting in a maths test and come across the following question:

Q4. Solve the following system of equations

$$\frac{x - 6}{3} = 4 + y$$

$$2x + 7y = 32$$

You are completely stuck so you sneak a glance at your smart friend's sheet. He's got the solution $(x=12, y=-2)$, is he right?

Checking Solutions - Example

Solution

(12, -2) is “ordered pair” solution. It means $x=12$, $y=-2$.

Substituting into equation 1:

$$LHS = \frac{12 - 6}{3} = 2$$

$$RHS = -2 + 4 = 2$$

LHS=RHS, so the values are a solution to equation 1.

Substituting into equation 2:

$$2(12) + 7(-2) = 24 - 14 = 10 \neq 32$$

Therefore LHS \neq RHS for equation 2.

Hence (12, -2) is not a solution.

Lesson learnt: Don't copy!

Inequality

Equations and variables don't always have a definite values, they can often take on a range of values.

Equation:

$$y = 3x + \frac{7}{x^2}$$

Inequality:

$$y > 2x^4$$



Greater Than



Greater Than or Equal To



Less Than



Less Than or Equal To

Inequality Types

There are four types of inequalities that can be used:

Less than, <

All numbers less than a determined value, for example $z < 3m$, all values of z are less than thrice m .

Greater than, >

All numbers greater than a determined value, for example $4x + 5 > y$, all values of the line $4x + 5$ are greater than y .

Inequality Types

Less than or equal to, \leq

Greater than or equal to, \geq

These are the same as less than and greater than except it includes the value. For example $n \geq 5$, means that n can be any number greater than 5, or 5 itself.

Sequences and Series

Sequence:

A set of numbers arranged in a definite order according to some definite rule.

Eg: $a_3, a_5, a_7, a_9, \dots$

Series:

The sum of terms in a sequence.

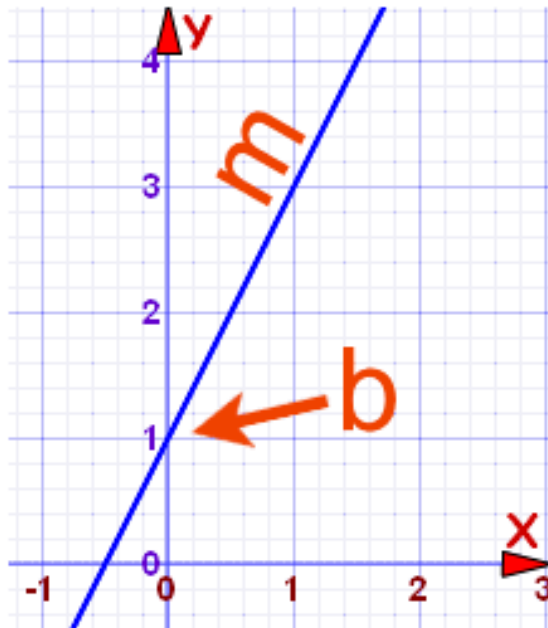
Eg: $a_1 + a_2 + a_3 + \dots +$

Common Graphs

At this level of maths, there are certain functions that come up again and again. Being able to visually recognise and remember the form of these plots will be a very helpful tool to tackling some problems.

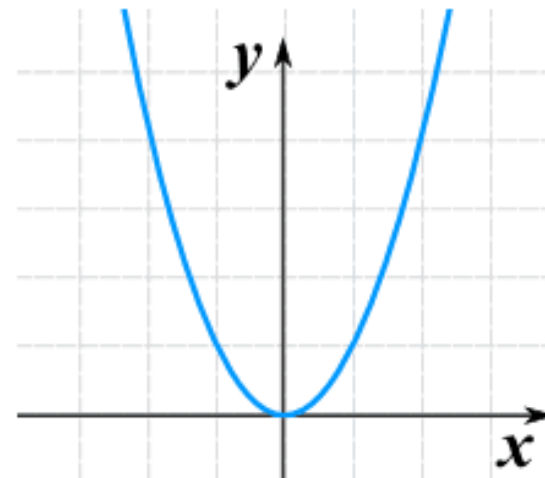
Common Graphs

Linear Function:



$$f(x) = mx + b$$

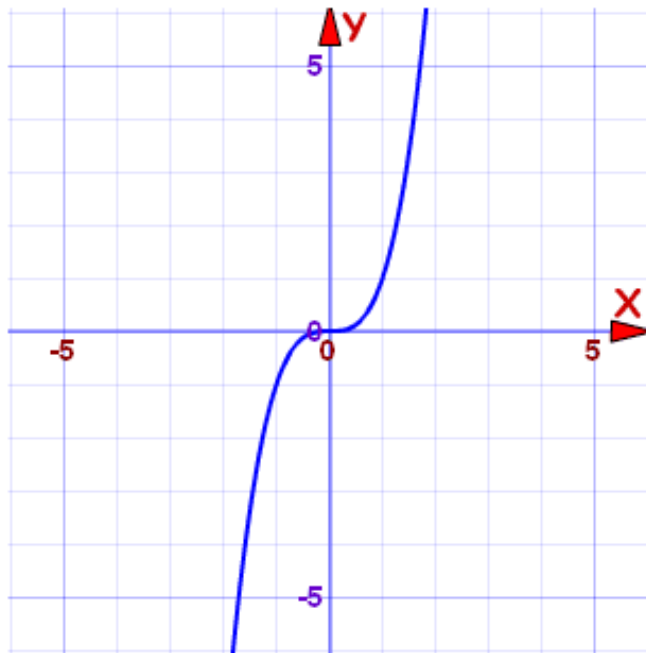
Square Function:



$$f(x) = x^2$$

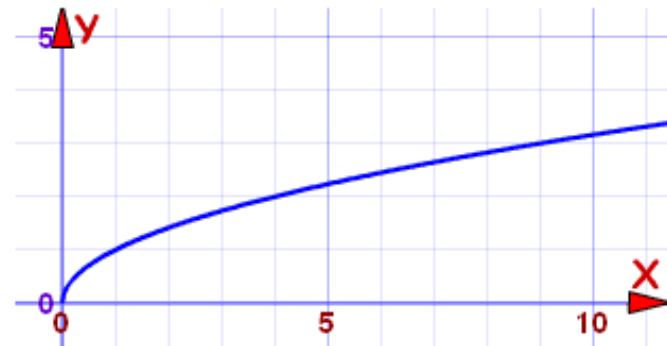
Common Graphs

Cube Function:



$$f(x) = x^3$$

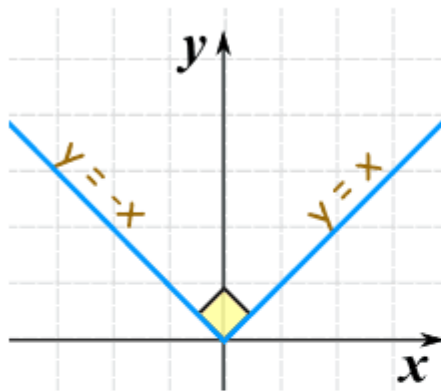
Square Root Function:



$$f(x) = \sqrt{x}$$

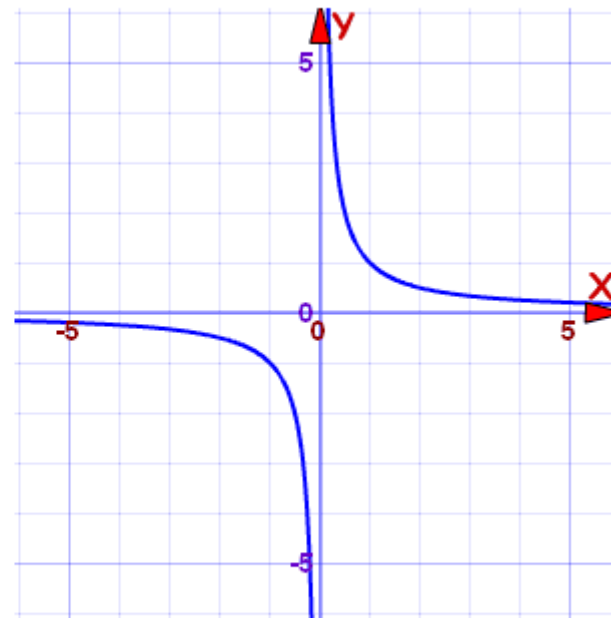
Common Graphs

Absolute Value Function:



$$f(x) = |x|$$

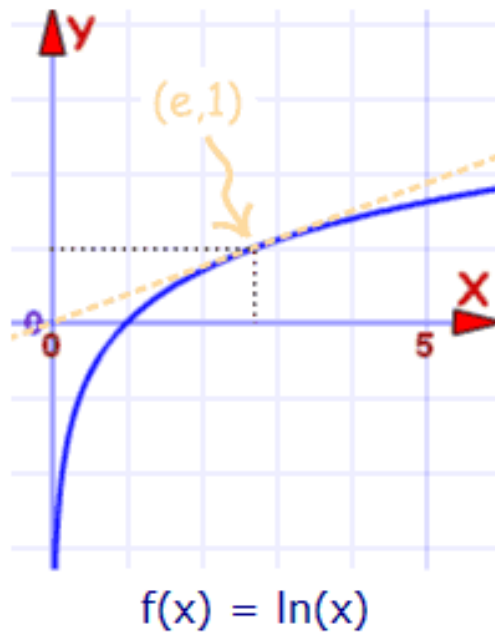
Reciprocal Function



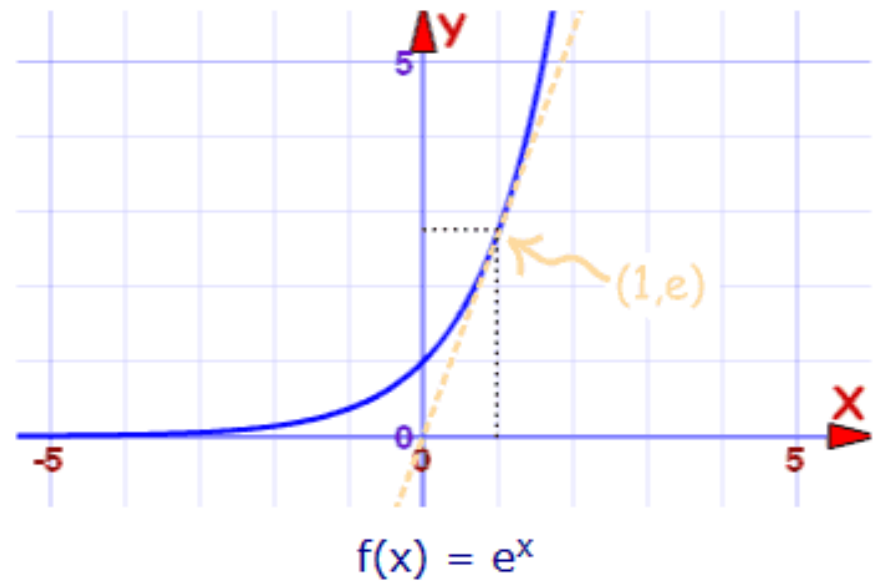
$$f(x) = 1/x$$

Common Graphs

Logarithmic Function:

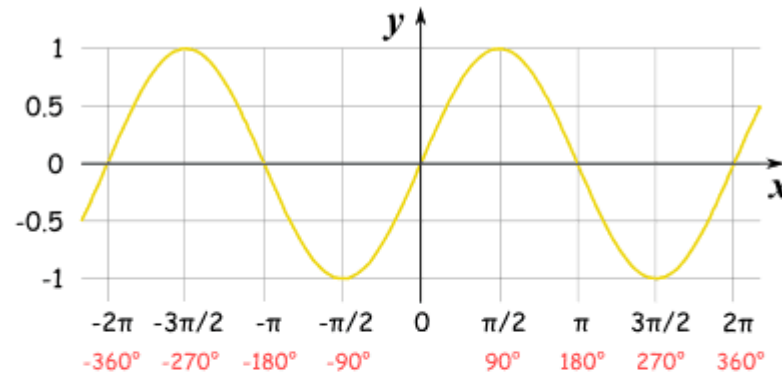


Exponential Function:

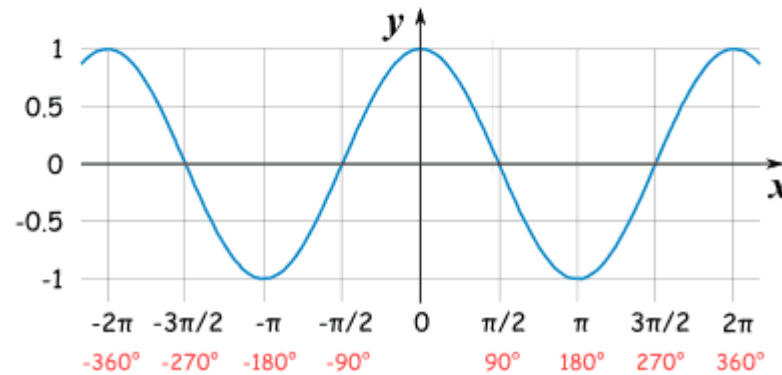


Common Graphs

Sine Function:



Cosine Function:



Common Graphs

But how do you remember all of them?



Trigonometry

Trigonometry is a branch of mathematics that studies triangles and the relationships between their sides and the angles between sides.

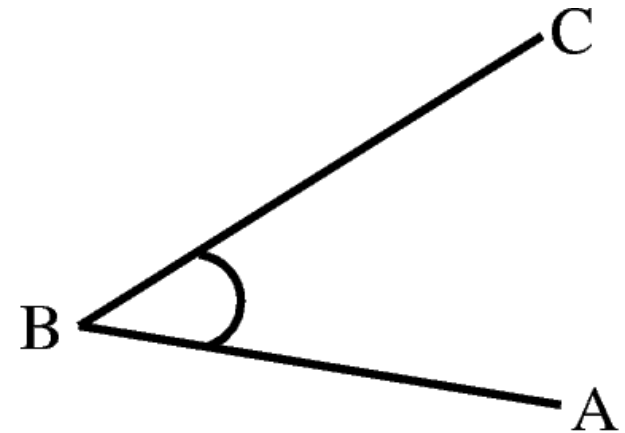
It has applications in both pure mathematics and applied mathematics, where it is essential in many branches of science and technology and is important in astronomy and satellite navigation.

Trigonometry is a methodology for finding some unknown elements of a triangle (or other geometric shapes).

Angle

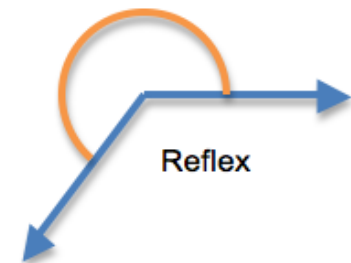
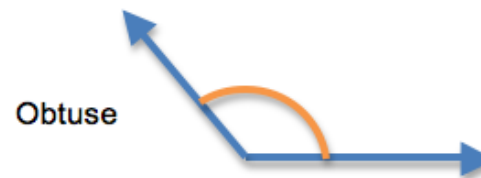
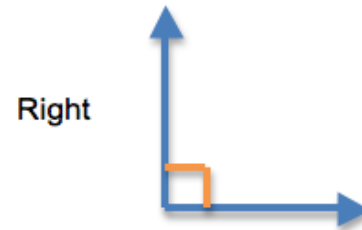
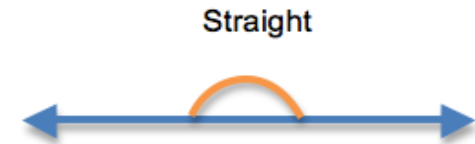
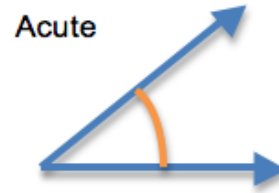
- Two straight lines commonly joined at the vertex
- The angle is named by letters corresponding to each arm and a central letter to name the vertex

$$\begin{aligned}\angle ABC &= \angle CBA \\ &= \angle B\end{aligned}$$



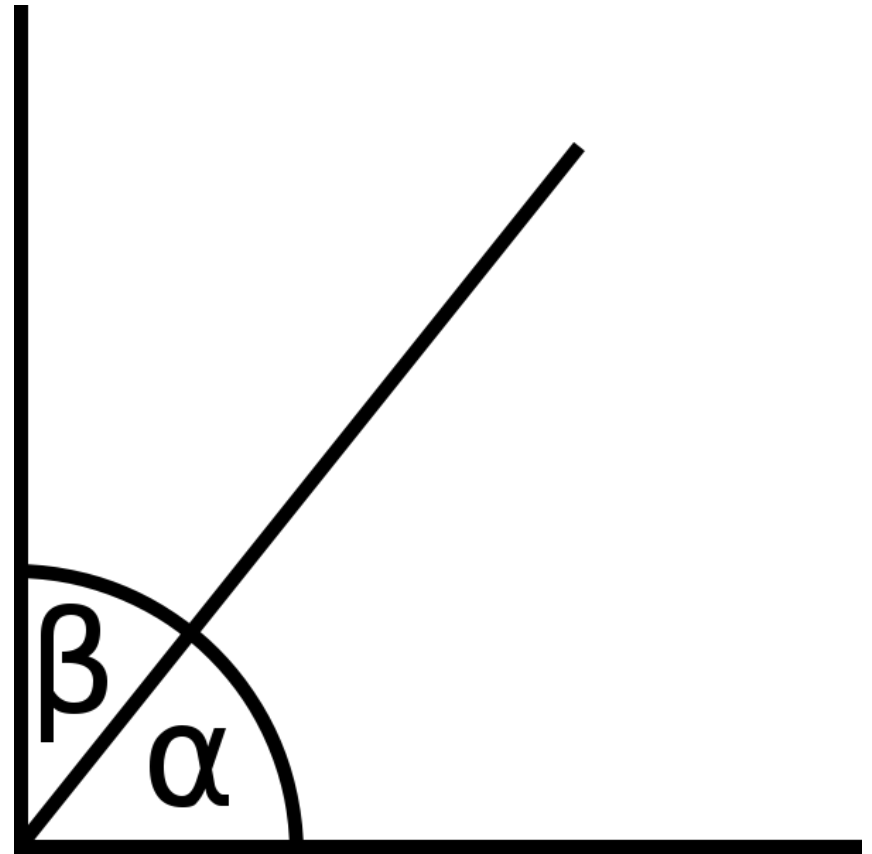
Types of Angles

- Acute: less than 90°
- Right: equal to 90°
- Obtuse: between 90° and 180°
- Straight: equal to 180°
- Reflex: between 180° and 360°



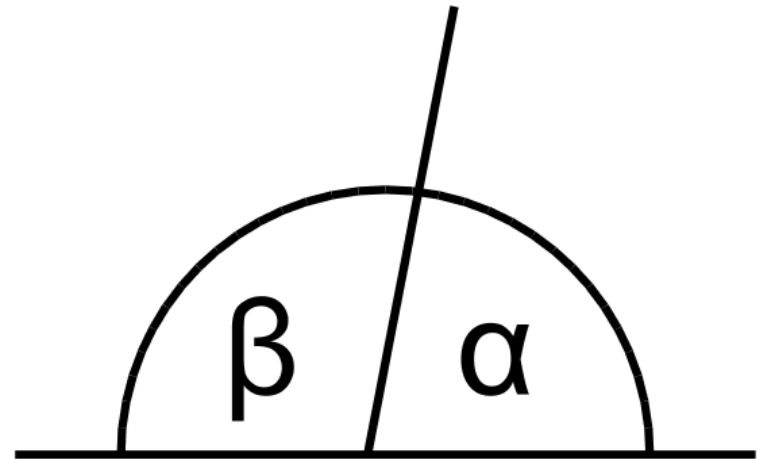
Complementary Angles

- Two or more adjacent angles that sum to 90°



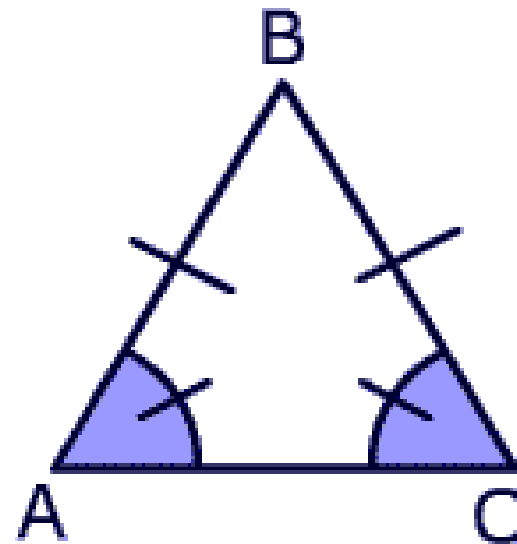
Supplementary Angles

- Two or more adjacent angles that sum to 180°



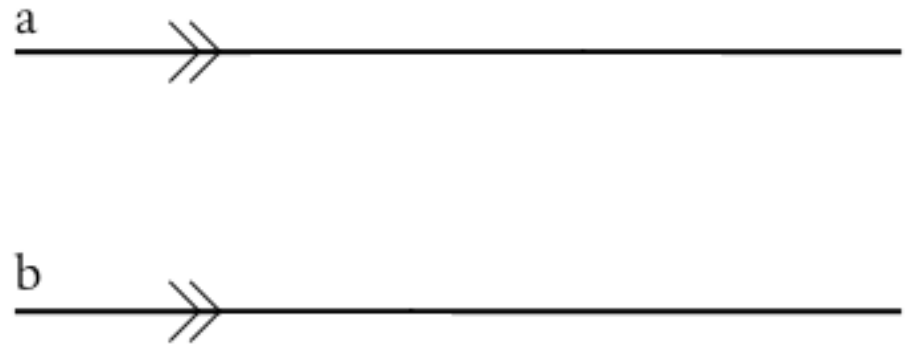
Congruent Angles

- Angles that are the same size and are indicated by the same symbol



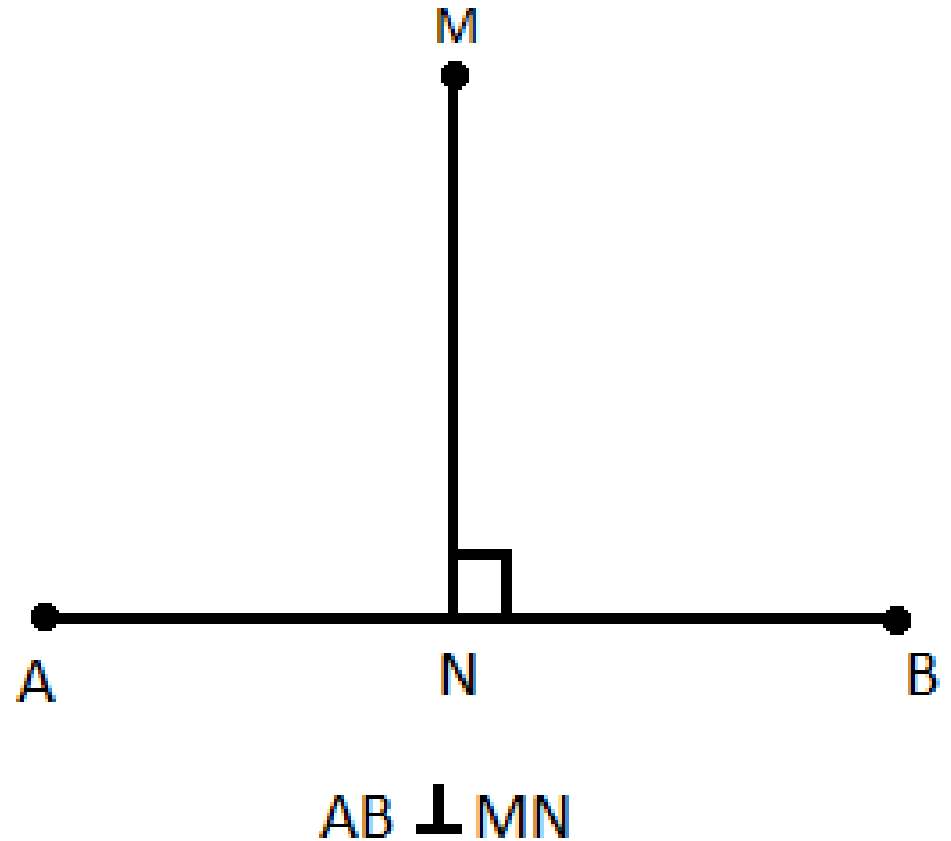
Parallel Lines

- Share no common point
- Parallel lines will never intersect
- Double or single arrows indicate that lines are parallel
- The \parallel symbol indicates two lines are parallel



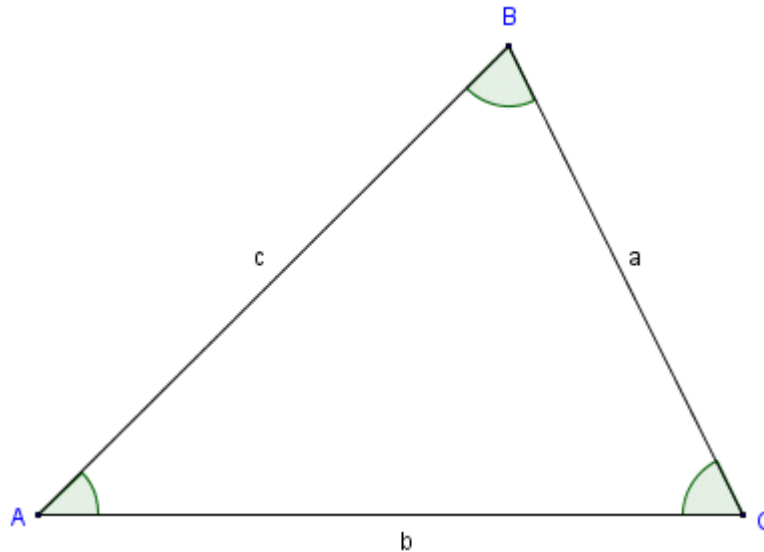
Perpendicular Lines

- Lines that intersect at right angles



Triangle Definitions and Labelling

Vertices of triangles are usually labelled with capitals whilst the edges are labelled with lower case letters.



And edge corresponds to the vertex directly opposite to it.

Triangle Definitions and Labelling

Angles of a triangle are usually denoted with Greek letters, typically the following are used:

α (alpha)

β (beta)

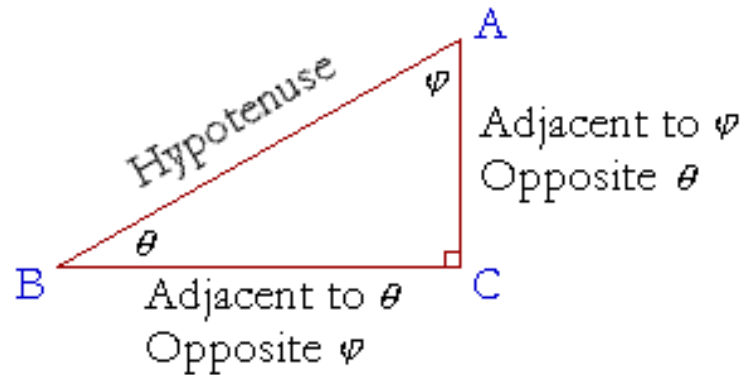
δ (delta)

θ (theta)

ϕ (phi)

Right Triangle

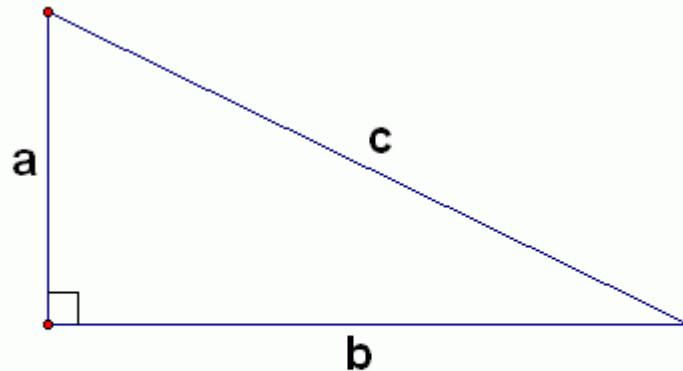
Right triangles have one angle that is equal to 90° .



The longest side of a right triangle is known as the **hypotenuse**. A side opposite a specified angle is called the **opposite** side. The side making up the angle that is not the hypotenuse is known as the **adjacent** side.

Pythagoras' Theorem

The square of the length of the longest side is equal to the sum of the squares of the two remaining sides.



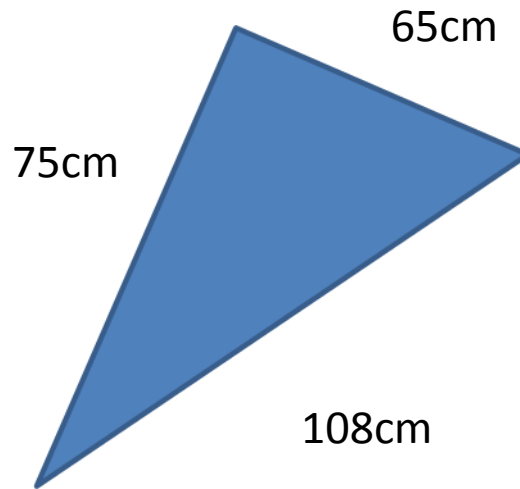
$$a^2 + b^2 = c^2$$

Using this formula, one can find the length of any side of a right-triangle if two are known.

Pythagoras' Theorem

This relationship only exists for right triangles, it can therefore be used to determine whether or not a triangle contains a right angle.

Example:



$$65^2 + 75^2 = 9850$$

$$108^2 = 11664$$

$$\therefore 65^2 + 75^2 \neq 108^2$$

Triangle is **not** a right triangle

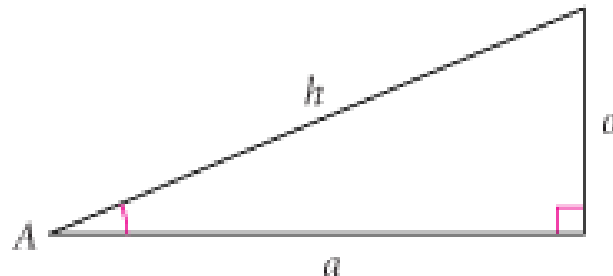
Trigonometric Ratios

The trigonometric ratios of a right angle triangle for the fundamental basis for all of trigonometry. These ratios are depicted below.

$$\text{sine of angle } A = \frac{\text{length of opposite leg}}{\text{length of hypotenuse}} \quad \text{or} \quad \sin A = \frac{o}{h}$$

$$\text{cosine of angle } A = \frac{\text{length of adjacent leg}}{\text{length of hypotenuse}} \quad \text{or} \quad \cos A = \frac{a}{h}$$

$$\text{tangent of angle } A = \frac{\text{length of opposite leg}}{\text{length of adjacent leg}} \quad \text{or} \quad \tan A = \frac{o}{a}$$



Common angles

- Common angles can be expressed in radians and left in exact values
 - Defined in terms of Pi

Degrees	Radians	Degrees	Radians
0	0	90	$\frac{\pi}{2}$
30	$\frac{\pi}{6}$	180	π
45	$\frac{\pi}{4}$	270	$\frac{3\pi}{2}$
60	$\frac{\pi}{3}$	360	2π

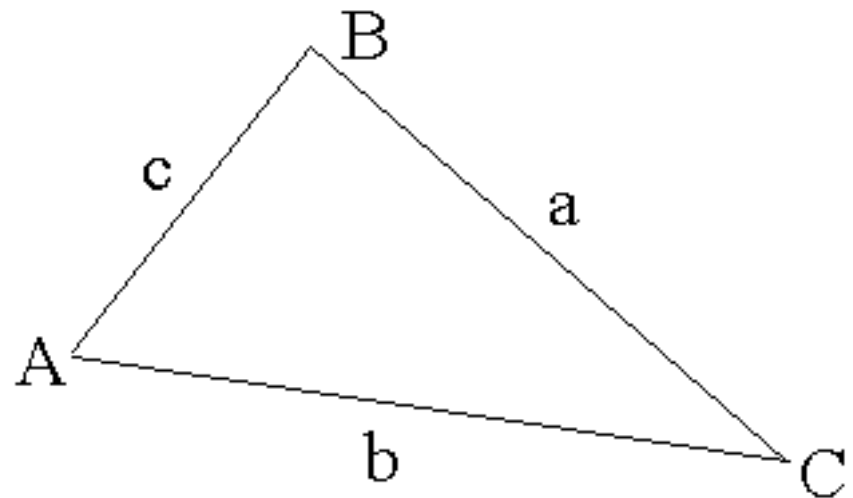
Sine Rule

- The Sine Rule can be applied to find any missing side and angle:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



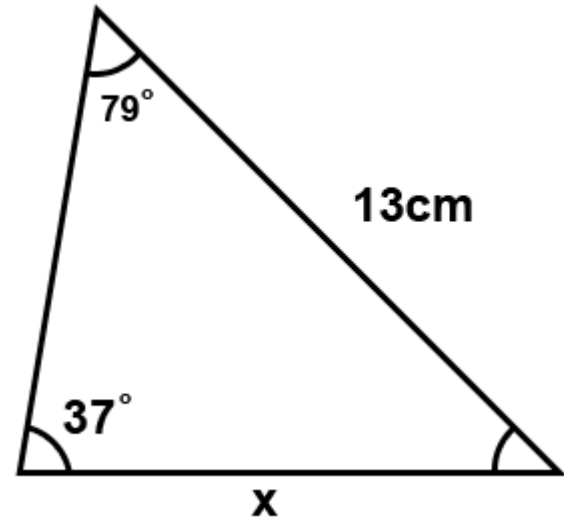
Sine Rule

- The sine rule can be applied to triangles that meet the following criteria:
- The values for 2 angles and 1 side are given
- The values of 2 sides and the value of the opposite angle is given

Example:

- Using the sine rule: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$$\frac{\sin 37}{13} = \frac{\sin 79}{x}$$
$$\Rightarrow x = \frac{13 \sin 79}{\sin 37}$$



Terminology and Definitions

Probability: Likelihood of something occurring

Experiment: An action where the result is uncertain, for example tossing a coin or rolling a dice.

Outcome: What can occur, for example a 5 of clubs.

Sample Space: Collection of all possible outcomes

Event: A single result of an experiment, for example rolling 5.

How to give a probability

A probability must be greater than or equal to zero, and less than or equal to 1.

Probabilities can be expressed as a fraction, decimal or percentage.

Example:

$$P(E) = \frac{3}{4} = 0.75 = 75\%$$

Phasors

- A phasor is a representation of a sinusoidal function where the amplitude (A), frequency (ω) and phase (θ) are all constant with time.
- From Euler's formula, a sinusoidal function can be represented as the sum of two complex functions

$$A \cos(\omega t + \phi) = A \left(\frac{1}{2} e^{i(\omega t + \phi)} + e^{-i(\omega t + \phi)} \right)$$

Phasors

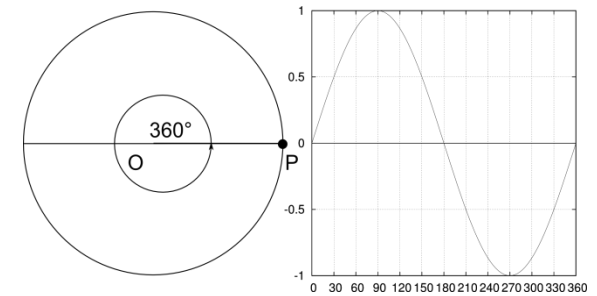
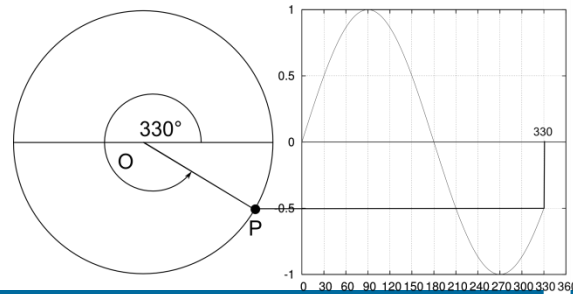
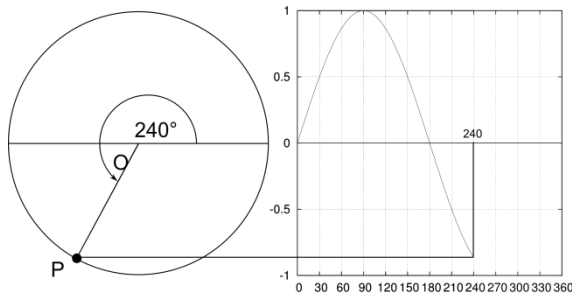
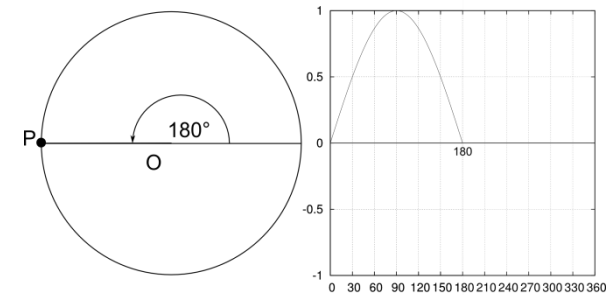
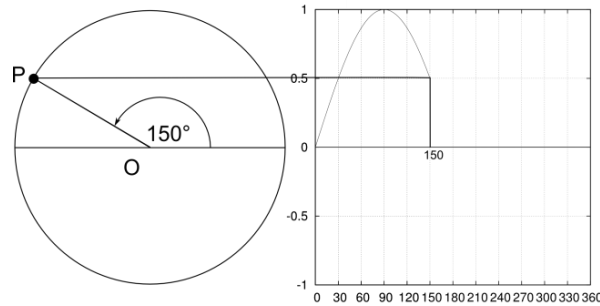
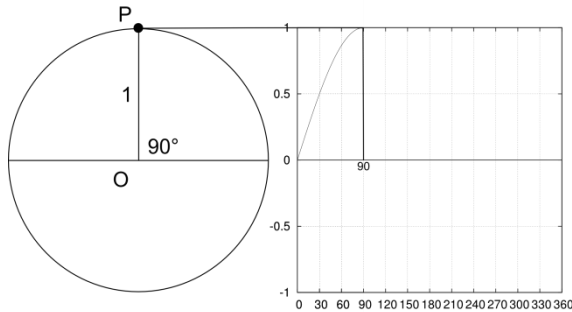
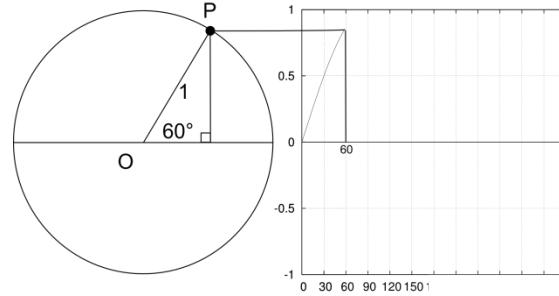
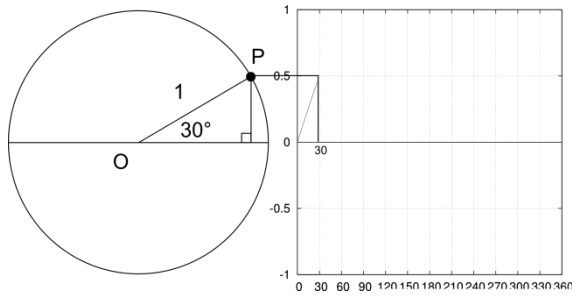
The second representation of the phasor is used to represent the amplitude and phase of the sinusoidal waveform

$$r \angle \theta$$

Rotating Vector is a "phasor"

- Way of representing sine waves
- Has frequency (how fast it spins)
- Has amplitude (how long it is)
- Has phase (relationship with another)

Phasors



Financial Math

Purchasing by Installment is buying an item using an Installment Loan, which you repay in installments.

The terms used in this category:

- a) Fixed Installment Loan
- b) Cash price of the item to be purchased
- c) Down Payment is an amount of money that you can to the seller at the time of purchase.

The things you need to calculate

- a) Amount financed or the amount you loan out from the seller.
(Cash price – down payment)
- b) Total Installment Price-The total value of the item including total monthly payment + down payment (It is the total money you pay to the seller at the end)
- c) Finance Charge – It is the financial burden you have to bear by way of interest
(Total installment price-Cash purchase price)
- d) Annual Percentage Rate (APR) –It is the interest rate that you paid on an installment loan

Purchasing by Installment

Example:

You wish to buy a pick-up truck, the cost of which is \$ 10,000 on cash. But you are unable to arrange the payment in full, but opted to go for a installment plan. The truck dealer is happy to suggest you that they can finance the truck by paying \$300 as down payment and \$200 per month for 60 monthly installments.

You will have to find out the amount financed, total installment price, finance charge and APR.

$$\begin{aligned}\text{Amount financed} &= \text{Cash price} - \text{down payment} \\ &= \$ 10,000 - \$ 300 \\ &= \$ 9,700\end{aligned}$$

Total installment price

payment

$$\begin{aligned}&= \text{Total monthly installment paid for 60 months} + \text{Down} \\ &= \$ 200 \times 60 \text{ months} + \$ 300 \\ &= \$ 12,000 + \$ 300 \\ &= \$ 12,300\end{aligned}$$

Finance charge = total installment price – cash price

$$= \$ 12,300 - \$ 10,000 = \$ 2,300$$

APR

Finance charges per \$100 financed

$$\begin{aligned}&= \text{Finance charge} \div \text{Amount financed} \\ &= \$ 2,300 \div \$ 9,700 \\ &= 0.237\end{aligned}$$

Percentage interest rate

$$= 23.7\%$$

Then, APR is calculated by multiplying the periodic interest rate by the number of periods in a year.

$$= 23.7\% \times 12 = 284.4\%$$

Corrected

Simple Interest

When L (lender) lends money to B (Borrower), the borrower usually pays a fee to the lender. This fee is called 'interest'. '**Simple**' interest, or '**flat rate**' interest. The amount of simple interest paid each year is a fixed percentage of the amount borrowed or lent at the start.

The simple interest formula is as follows:

$$\text{Interest} = \text{Principal} \times \text{Rate} \times \text{Time}$$

where:

'**Interest**' is the total amount of interest paid,

'**Principal**' is the amount lent or borrowed,

'**Rate**' is the percentage of the principal charged as interest each year. The rate is expressed as a decimal fraction, so percentages must be divided by 100. For example, if the rate is 15%, then use 15/100 or 0.15 in the formula.

'**Time**' is the time in years of the loan.

Simple Interest

The simple interest formula is often abbreviated in this form:

$$I = P R T$$

Three other variations of this formula are used to find P, R and T:

$$P = \frac{I}{RT}, \quad R = \frac{I}{PT}, \quad T = \frac{I}{PR}$$

Simple interest problems can involve lending or borrowing. In both cases the same formulas are used. Whenever money is borrowed, the total amount to be paid back equals the principal borrowed plus the interest charge:

Total repayments = (principal + interest)

Simple Interest

Example 8.3.2.1:

A student purchases a computer by obtaining a simple interest loan. The computer costs \$1500, and the interest rate on the loan is 12%. If the loan is to be paid back in weekly installments over 2 years, calculate:

1. The amount of interest paid over the 2 years,
2. The total amount to be paid back,
3. The weekly payment amount.

Here, The principal: 'P' = \$1500, **interest rate:** 'R' = 12% = 0.12, **repayment time:** 'T' = 2 years

Find the amount of **interest** paid.

$$\begin{aligned}\text{Interest: } I &= PRT \\ &= 1500 \times 0.12 \times 2 \\ &= \$360\end{aligned}$$

Find the total amount to be paid back.

$$\begin{aligned}\text{Total repayments} &= \text{principal} + \text{interest} \\ &= \$1500 + \$360 \\ &= \$1860\end{aligned}$$

Calculate the weekly payment amount

$$\begin{aligned}\text{Weekly payment amount} &= \frac{\text{Total repayments}}{\text{Loan period, T, in weeks}} \\ &= \frac{\$1860}{2 \times 52} = \$17.88 \text{ per week}\end{aligned}$$

Compound Interest

Compound interest calculations apply to investments where the amount of interest is calculated on the present balance of the account. The amount you invest is called the '**Principal**'.

Example 8.3.3.1:

If you invest a principal of \$1000 at 10% compound interest paid monthly, then after the first month, the interest payment will be:

$$\text{Interest (first month)} = 10\% \text{ of } \$1000 = \$100$$

If the interest is added to the principal, you now have: $\$1000 + \$100 = \$1100$, so the next month's interest will be 10% of the new total:

$$\text{Interest (second month)} = 10\% \text{ of } \$1100 = \$110$$

So the principal increases to \$1210 after the second month. Notice that the increase is \$10 greater after the second month than after the first. This trend will continue during the life of the investment so that it will continue to grow faster and faster as time goes on.

The compound interest formula calculates the value of a compound interest investment after 'n' interest periods.

$$A = P (1 + I)^n$$

where:

'A' = Amount after 'n' interest periods.

'P' = Principal, the amount invested at the start.

'I' = Interest rate applying to each period.

'n' = Number of interest periods

Compound Interest

Using this formula to duplicate the results from the example above:

$$P = \$1000, I = 0.1, n = 2,$$

$$\text{so: } A = 1000(1 + 0.1)^2 = 1000 \times 1.21 = \$1210$$

The interest rate is per interest period. Often, interest rates are given for a whole year, (per annum). A yearly interest rate must be divided by the number of payments per year.

For example, if the interest rate on an investment is 12% per annum, and interest is paid monthly, then the value of 'I' to use in the formula is $12\%/12 = 1\% = 0.01$

By rearranging the compound interest formula, we can use it to find unknown 'P', 'I' and 'n' values, like this:

Example 8.3.3.2:

For example: What principal does Andrew need to invest at 15% p.a. compounding monthly so that he ends up with \$10000 at the end of 5 years?

$$\text{Here, } A = \$10,000, I = 0.15, n = 5$$

Use the formula

$$P = \frac{A}{(1 + I)^n} \quad P = \frac{10,000}{(1 + 0.15)^5} = \frac{10,000}{2} = \$5,000$$

Depreciation

Depreciation is the method of assigning costs to the wear and tear of tangible fixed assets over its estimated useful life. Depreciation is an accounting system where accountants decrease the value of the asset over the years based on its usefulness. As understood by layman, depreciation is the decline in the value of a particular asset as the years pass by.

Depreciation Methods

All depreciation methods have the following general formula:

$$\text{Depreciation Expense} = \text{Depreciation Base} \times \text{Depreciation Rate}$$

Types of Depreciation Method

1. Straight-line method
2. Accelerated Methods
 - a) Sum of the year's digits method
 - b) Double-declining-balance method

Depreciation

Straight Line Depreciation Method

The straight line depreciation is calculated by taking the purchase or acquisition price of an asset subtracted by the salvage value divided by the total productive years the asset can be reasonably expected to benefit the company (called "useful life" in accounting jargon).

Straight Line Depreciation Calculation

(Purchase Price of Asset - Approximate Salvage Value) ÷
Estimated Useful Life of Asset

Depreciation

Example 8.3.4.1:

A new computer is bought at a cost of \$5,000.

A salvage value of \$200 selling parts when it is disposed

Allow a maximum useful life of three years for computers.:

$(\$5,000 \text{ purchase price} - \$200 \text{ approximate salvage value}) \div 3 \text{ years estimated useful life}$

The answer, \$1,600, is the depreciation charges annually, using the straight line method.

Depreciation Base	Depreciation Rate
Cost - Salvage value	1/ Estimated useful life
\$ 5000 - \$ 200 = \$ 4800	1/3 = 33.33%

Depreciation

Accelerated Depreciation-Sum of the year's digits method

To calculate depreciation charges using the sum of the year's digits method, take the expected life of an asset (in years) count back to one and add the figures together.

10 years useful life = $10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$ Sum of the years = 55

In the first year, the asset would be depreciated $10/55$ in value (the fraction $10/55$ is equal to 18.18%), $9/55$ (16.36%) the second year, $8/55$ (14.54%) the third year, etc.

Depreciation

Example 8.3.4.2:

A new computer is bought at a cost of \$5,000.

A salvage value of \$200 selling parts when it is disposed

Allow a maximum useful life of 3 years for computer.

3 years useful life = 3 + 2 + 1 Sum of the years = 6

Taking \$5,000 - \$200 we have a depreciation base of \$4,800.

1. In the first year, the computer would be depreciated by 3/6ths (50%), --\$ 2,400
2. In the second year, by 2/6 (33.33%) -----\$ 1,599.84
3. In the third and final year by the remaining 1/6 (16.67%). -----\$ 800.16

Note: The straight-line example would have simply charged \$1,600 each year, distributed evenly over the three years useful life.